

# Determining neutrino mass hierarchy from electron disappearance at a Low energy neutrino factory

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## Abstract

Recent measurements of large  $\theta_{13}$  by Daya Bay and RENO reactor experiments have opened up the possibility of determining the neutrino mass hierarchy, i.e. the sign of the mass squared splitting  $\Delta m_{31}^2$ , the CP-violating phase  $\delta_{CP}$  and the octant of  $\theta_{23}$ . In the light of this result, we study the performance of a low energy neutrino factory (LENF) for determination of the mass hierarchy. In particular, we explore the potential of the  $\nu_e$  and  $\bar{\nu}_e$  disappearance channels at LENF to determine the neutrino mass hierarchy, that is free from the uncertainties arising from the unknown  $\delta_{CP}$  phase and the  $\theta_{23}$  octant. We find that using these electron neutrino (antineutrino) disappearance channels with a standard LENF, it is possible to exclude the wrong hierarchy at  $5\sigma$  with only 2 years of running.

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## I. INTRODUCTION

The discovery of flavour mixing of atmospheric and solar neutrinos in the golden years of neutrino oscillations (1998-2004) has led to extensive theoretical and experimental effort in neutrino physics, worldwide. Precision measurements of flavour mixing parameters in the lepton sector opens up yet another window in our quest for physics beyond the Standard Model.

Neutrino oscillation phenomena are described in terms of six independent parameters namely, three mixing angles ( $\theta_{23}$ ,  $\theta_{13}$ ,  $\theta_{12}$ ), two mass squared differences ( $\Delta m_{31}^2$ ,  $\Delta m_{21}^2$ ) and a CP-violating phase ( $\delta_{CP}$ ). In the last decade, data from solar and reactor neutrino experiments have resulted in information on the sign and magnitude of  $\Delta m_{21}^2$  and a precise value of  $\theta_{12}$  [1, 2]. The atmospheric parameters  $|\Delta m_{31}^2|$  and  $\theta_{23}$  have been measured and their precision will be increased by T2K [3] and NO $\nu$ A [4]. Recent measurements of  $\theta_{13}$  from T2K [5], MINOS [6], and Double Chooz [7] indicated a non zero value of  $\theta_{13}$ . This year, a moderately large value of  $\theta_{13}$  has been established by the reactor experiments. Daya Bay [8] claimed that  $\theta_{13}$  is non vanishing with a significance of 5.2 standard deviations while RENO [9] measured a larger central value but with a slightly lower significance of  $4.9\sigma$  coming from their higher systematic error. In fact, the global fits [10, 11] of neutrino oscillation parameters including these recent measurements exclude  $\sin^2 \theta_{13} = 0$  at  $10.2\sigma$ . Large  $\theta_{13}$  enables a wide range of possibilities for determination of the neutrino mass hierarchy, the value of  $\delta_{CP}$  and the octant of  $\theta_{23}$  – the remaining unknown neutrino oscillation parameters. In fact, a non zero value of  $\theta_{13}$  is a prerequisite to probe these unknowns.

Neutrino mass hierarchy or the neutrino mass ordering has profound theoretical implications. Till now we have been unable to resolve the neutrino mass hierarchy, i.e, whether the hierarchy is normal (NH,  $\Delta m_{31}^2 > 0$ ) or inverted (IH,  $\Delta m_{31}^2 < 0$ ). For neutrinos passing through long baselines, the effect on oscillation parameters in presence of earth matter [12–14] is dependent on the sign of  $\Delta m_{31}^2$ , hence such long baseline experiments can resolve the question of mass hierarchy. Moreover, the effect of the matter potential (enhancement or suppression) for each hierarchy is different for neutrinos versus antineutrinos, requiring charge identification of the final lepton produced in the interaction of the neutrinos (antineutrinos) with the detector material. These earth matter effects are enhanced if  $\theta_{13}$  is sufficiently large, improving the chances of discovery of mass hierarchy.

With the favourable large  $\theta_{13}$ , future atmospheric neutrino experiments will play a crucial role in determination of mass hierarchy through disappearance channels (muon and electron). Preliminary

studies of the India-based Neutrino Observatory (INO) [15] indicate that, INO with its charge identification capability of muons, will be able to make a measurement at  $2.7\sigma$  statistical significance, with 500 kton-year exposure [16]. Similar sensitivity will be possible at Hyperkamiokande [17] (with electron events) and with PINGU at Icecube [18], while better sensitivity will be achievable with a rather futuristic magnetized liquid Argon detector [19] with visibility of both muon and electron events. Accelerator long baseline experiments like T2K and NO $\nu$ A will have good sensitivity but only for some fraction of the  $\delta_{CP}$  range. In fact, recently in Ref. [20], a combined analysis of T2K and NO $\nu$ A has been performed and it is found that mass hierarchy can only be determined for  $-150^\circ \leq \delta_{CP} \leq -30^\circ$  if the true hierarchy is NH and  $30^\circ \leq \delta_{CP} \leq 150^\circ$  if it is IH, indicating the difficulty in achieving this without knowledge of the true  $\delta_{CP}$ , unless statistics is sufficiently high [21]. However, a detailed study of neutrino mass hierarchy performed in Ref. [22] has demonstrated that the use of atmospheric data from the proposed INO in conjunction with accelerator beam experiments T2K and NO $\nu$ A results in a  $3\sigma$  determination of hierarchy with a high resolution 100 kton detector for  $\sin^2 2\theta_{13} = 0.09$ . Also, in Ref. [23], the authors have shown that mass hierarchy can be determined for any value of  $\delta_{CP}$  from the near resonant matter effect in the  $\nu_\mu \rightarrow \nu_e$  oscillation channel at  $5\sigma$  with a superbeam with an average neutrino energy of 5 GeV at Super-Kamiokande that is at a distance of 8770 km from the proposed superbeam facility at CERN.

In this paper we wish to focus on the proposed low energy neutrino factory (LENF) for the mass hierarchy determination. In a neutrino factory both  $\nu_\mu$  ( $\bar{\nu}_\mu$ ) and  $\nu_e$  ( $\bar{\nu}_e$ ) beams are produced from the decay of muon ( $\mu^\pm$ ) in the long straight sections of a storage ring. A  $\mu^+$  decay produces  $\bar{\nu}_\mu$  and  $\nu_e$  whereas a  $\mu^-$  decays to  $\nu_\mu$  and  $\bar{\nu}_e$ . Hence a detailed study of various oscillation channels such as  $\nu_\mu \rightarrow \nu_e$ ,  $\nu_\mu \rightarrow \nu_\mu$ ,  $\nu_e \rightarrow \nu_\mu$ ,  $\nu_e \rightarrow \nu_e$  and all the corresponding antineutrino oscillations are possible using a neutrino factory. In addition there are the tau appearance channels, however, these can be utilized only in the presence of a far detector with tau lepton identification capability. Apart from the availability of large number of channels, this facility is preferred due to the feasibility of high beam intensity and accurate knowledge of the neutrino fluxes.

The so called ‘golden’ channel, i.e, the  $\nu_e \rightarrow \nu_\mu$  oscillation, contains information on all the oscillation parameters and hence in principle can be used to determine all the unknown parameters. However, degenerate solutions in the parameter space make this task difficult. Till recently, since the unknowns were:  $\theta_{13}$ , CP violating phase  $\delta_{CP}$ , the octant of  $\theta_{23}$  (if nonmaximal) and the mass hierarchy, this resulted in an eight fold parameter degeneracy [24, 25], making it hard to extract

unique values of the parameters. The solution to this problem was first provided in Ref. [25, 26]. They found that at the ‘magic’ baseline of around 7500 km, the  $\nu_e \rightarrow \nu_\mu$  oscillation probability does not depend on the CP phase  $\delta_{CP}$  irrespective of the beam energy. Indeed it would be possible to determine the mass hierarchy at the magic baseline and the high energy neutrino factory (HENF) would be the suitable facility for this long baseline study. The physics potential of a HENF has been studied by various authors [27] and a detail analysis of optimization of the neutrino factory baseline by the IDS-NF group suggests two baselines: one at the magic baseline  $L = 7000 - 8000$  km and the other at  $L = 2500 - 5000$  km with a muon beam energy of 25 GeV [28]. Recent studies however suggested that a LENF with a baseline of 1300 km (FNAL to DUSEL) with a muon beam energy of 4.5 GeV could achieve the desired precision if  $\theta_{13}$  is large enough [29]. A re-optimization of the neutrino factory resulted in the proposal for a staged approach [30], which may have the option of much lower (4 – 10 GeV) muon beam energies and shorter baselines (1000 – 2500 km). The LENF has already been shown to have excellent sensitivity to the mass hierarchy from the golden and the platinum,  $\nu_\mu \rightarrow \nu_e$  channels [31]. In Ref. [32], it has been shown that only if  $\sin^2 \theta_{13} > 10^{-2}$ , mass hierarchy can be determined at  $5\sigma$  independently of  $\delta_{CP}$  at muon beam energies more than 5 GeV and baselines longer than 1000 km, using the golden channel.

In this study, we show that, with the knowledge of  $\theta_{13}$  from the recent reactor experiments, determination of mass hierarchy can be made completely free of degeneracies by exploiting the electron disappearance channel. For electron disappearance ( $\nu_e/\bar{\nu}_e \rightarrow \nu_e/\bar{\nu}_e$ ), the oscillation probability  $P_{ee}$  for neutrinos (antineutrinos) of energy  $E_\nu$  traveling over a baseline of length  $L$  in the presence of matter potential  $V = \pm\sqrt{2}G_F n_e$ , can be expressed as a perturbative expansion in the parameters  $\alpha = \Delta m_{21}^2/\Delta m_{31}^2$  and  $\sin \theta_{13}$  as [33]:

$$P_{ee} = 1 - \alpha^2 \sin^2 2\theta_{12} \frac{\sin^2 A \Delta}{A^2} - 4 \sin^2 \theta_{13} \frac{\sin^2(A-1)\Delta}{(A-1)^2}, \quad (1)$$

where,  $\Delta = (\Delta m_{31}^2 L/4 E_\nu)$ ,  $A = (2 E_\nu V/\Delta m_{31}^2)$ ,  $G_F$  is the Fermi constant and  $n_e$  is the electron density. The sign of the potential  $V$  is positive for neutrinos and negative for antineutrinos. From Eq. 1, one can see that since this oscillation channel is independent of  $\delta_{CP}$  and  $\theta_{23}$ , it is free from the degeneracies arising from the unknown octant of  $\theta_{23}$  and from the lack of knowledge of the CP violating phase. Hence, at least in principle, it is possible to use this channel to determine the hierarchy independently of both these parameter values.

Previous studies of the neutrino factories performance had ignored this channel. The reason

being that one assumed that  $\theta_{13}$  is perhaps very small. Mass hierarchy sensitivity of a setup was then judged by the smallest value of  $\sin^2 2\theta_{13}$  above which the wrong hierarchy could be excluded. In  $P_{ee}$ , since the mass hierarchy sensitive term is quadratic in  $\sin \theta_{13}$ , therefore one did not expect much sensitivity for small reactor angles. On the other hand in the golden channel while there is one atmospheric (mass hierarchy sensitive) term appearing with  $\sin^2 2\theta_{13}$  dependence and could dominate only for large  $1 - 3$  mixing angle, but in addition, there is also an interference term which is linear in  $\sin 2\theta_{13}$  which allows mass hierarchy sensitivity for intermediate values also. For very small  $\theta_{13}$ , sensitivity was expected only with very high intensity neutrino beams and long baselines to enhance the matter effect and hence HENFs of 50 GeV beam energies and rather long baselines were considered. Feasibility of mass hierarchy determination with these facilities was shown even for as small values as  $\sin^2 2\theta_{13} \leq 10^{-4}$ . In case of muon disappearance, there are many terms which are sensitive to mass hierarchy, with coefficients independent of  $\theta_{13}$  but quadratic in solar mass squared difference, ones that depend on  $\sin^2 \theta_{13}$  as well as those which are linear in  $\sin \theta_{13}$  and the solar mass squared difference. With many terms of differing signs, for a small value of the reactor angle, sensitivity was possible again only by going to long baselines and high energy intense beams, while with large values it is feasible even with atmospheric neutrinos as described earlier. Apart from the dependence on the  $1 - 3$  mixing angle, the sensitivity to systematics for the electron disappearance channel also had a role to play. Any experiment that measures events coming from  $P_{ee}$ , a disappearance channel, has to detect a small deficit in the expected number of neutrino events. The extent of the deficit depends of course on the value of  $\theta_{13}$ . In the light of recent measurements indicating a large value of  $\theta_{13}$ , we claim that it is possible to extract information about the hierarchy from this channel.

Focussing in this paper on the electron disappearance channel at LENF, we demonstrate that for a large value of  $\theta_{13}$  (Daya Bay range) it is possible to exclude the wrong hierarchy at a  $5\sigma$  for all values of  $\delta_{CP}$  and any octant of  $\theta_{23}$ , irrespective of the choice of the true hierarchy. Hence unlike all other oscillation channels, this channel can be used for a clean determination of the mass hierarchy not just at some magic baselines, but rather for all baselines greater than about 1200 km corresponding to muon beam energies about 3 GeV or larger. Moreover, this channel can give us an independent confirmation of hierarchy measurement from the golden channel. The possibility of using electron disappearance channel to study mass hierarchy has been studied in Ref. [34–37] in the context of a  $\beta$ -beam as a  $\nu_e$  source, however, this requires rather high boost power.

After the recent measurement of a large  $\theta_{13}$  value, the possibility of determining mass hierarchy with electron disappearance with reactor neutrinos has also been investigated again by several authors. Authors of Ref. [38] have concluded that such a measurement will be difficult due to the finite detector energy resolution, while Ref. [39] have described the challenges and possible solutions. In Ref. [40] it is pointed out that with rather large exposures,  $3\sigma$  mass hierarchy discrimination seems feasible.

Our paper is organized as follows. In section II, we start with a brief description of our experimental setup and then provide a detail description of the numerical simulations. The results are reported in section III with a conclusion in section IV.

## II. DETECTOR SETUP AND SIMULATIONS

With the large value of  $\theta_{13}$  confirmed independently by Daya Bay and RENO reactor experiments, the LENF is a good facility to determine the mass hierarchy. A LENF with a baseline of 1300 km (FNAL to DUSEL) and a muon beam energy of 4.5 GeV was first proposed in Ref. [29]. It was shown that for sufficiently high statistics and detection efficiency, an optimized LENF can be an excellent set up for precision measurements of oscillation parameters for a large value of  $\theta_{13}$ . In the context of LENF, two types of detector technologies have been discussed in the literature: a 20 kton magnetized totally active scintillator detector (TASD) and a 100 kton liquid argon detector (LAr) with charge identification capabilities of both electrons and muons.

In our study, we consider an LENF setup of Ref. [41] with a magnetized 20 kton TASD with an energy resolution of 10% for all channels and a lower energy threshold of 0.5 GeV. The detection efficiency of the TASD is 37% below and 47% above 1 GeV for electron events with a background at the  $10^{-2}$  level. We use a LENF with  $1.4 \times 10^{21}$  useful muons per year per polarity and a running time of 2 years. In our initial analysis we use a baseline of 1300 km and muon beam energy  $E_\mu = 4.5$  GeV. For more details on low energy neutrino factory we refer to Ref. [41].

Our goal is to investigate the performance of the above LENF setup in determining the neutrino mass hierarchy using electron disappearance channel, as this channel has the advantage of being independent of  $\delta_{CP}$  and  $\theta_{23}$ . Earlier, with  $\theta_{13}$  unknown (and hence possibly very tiny), most studies used to look for the hierarchy reach in  $\sin^2 2\theta_{13}$ , defining it as the limiting value of  $\sin^2 2\theta_{13}$  above which the wrong hierarchy could be excluded at a chosen confidence level. Now that  $\sin^2 2\theta_{13}$  has

been measured to be fairly large with good precision, there is no point in trying to find the minimum  $\sin^2 2\theta_{13}$  for which the wrong hierarchy can be distinguished. Instead, for most of our analysis, we use the central value of  $\sin^2 2\theta_{13}$  from RENO, as well as the value corresponding to  $2\sigma$  lower limit of  $\sin^2 \theta_{13}$  coming from the global fit [10] and see if the electron disappearance channel alone can achieve the desired precision.

For our numerical simulations, we use the following true values of the oscillation parameters: for the leading atmospheric parameters, we use  $\theta_{23} = 45^\circ$  and  $\Delta m_{\text{eff}}^2 = 2.4 \times 10^{-3} \text{ eV}^2$ , for the solar parameters, we use  $\sin^2 \theta_{12} = 0.304$  and  $\Delta m_{21}^2 = 7.65 \times 10^{-5} \text{ eV}^2$ . For the third mixing angle  $\theta_{13}$ , we use the central value of  $\sin^2 2\theta_{13} = 0.113$  from RENO. The unknown CP violating phase  $\delta_{CP}$  is varied over its full range  $-180^\circ$  to  $180^\circ$ . Here  $\Delta m_{\text{eff}}^2$ , an effective mass-squared difference measured in  $\nu_\mu$  survival probability, is related to  $\Delta m_{31}^2$  via [42]

$$\Delta m_{\text{eff}}^2 = \Delta m_{31}^2 - (\cos^2 \theta_{12} - \cos \delta \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23}) \Delta m_{21}^2. \quad (2)$$

To evaluate the event rates for production of electrons in the detector, we use the differential neutrino factory flux,  $\Phi_i \equiv dN_i/dE_\nu$ , where  $i = e, \mu$ <sup>1</sup> corresponds to flavour of neutrinos, double differential cross-section  $\sigma_e \equiv d^2\sigma_e dE_e/d\cos\theta_e$  for CC interactions (quasi-elastic, resonance and deep inelastic processes) producing an electron (positron) in the detector and the kinematic constraints that have been described in detail in Ref. [43, 44]. The event rates for production of electrons (positrons) in the detector are defined as

$$\mathcal{N}_{ie} = \kappa \int \Phi_i P_{ie} \sigma_e(\nu_e \rightarrow e) \epsilon_e,$$

where  $\kappa$  accounts for the exposure (size of detector and years of running),  $\epsilon_e$  are the detection efficiencies and the oscillation probability  $P_{ie}$  is a function of the energy  $E_\nu$  of the neutrino and the length of the baseline traveled by the neutrino, before reaching the detector. The integration is over all the relevant variables, including resolution function, corresponding to bins in the observed lepton energy,  $E_e^{\text{obs}}$ .

For our statistical analysis, we use a gaussian  $\chi^2$  with the events coming from both  $\nu_e$  and  $\bar{\nu}_e$  disappearance channels. We generate our experimental data for a fixed or true hierarchy keeping all other parameters fixed. The theoretical data is then generated using the other, wrong hierarchy. The resulting  $\chi^2$  thus determines the confidence level at which the wrong hierarchy can be excluded.

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<sup>1</sup> while we use the e-disappearance channels alone for the signal events, however, background events appear from the platinum channel also, hence the initial muon flux also needs to be considered.

However, for realistic analysis one needs to marginalize over all the parameters in order to include the uncertainties coming from them. Thus the minimum  $\chi^2$  is determined after taking all the variations into account. For our analysis, priors are used for the measured oscillation parameters. An error of 10% is taken for the atmospheric parameters  $\sin^2 2\theta_{23}$  and  $|\Delta m_{\text{eff}}^2|$  and 4% on the solar parameters  $\sin^2 \theta_{12}$  and  $\Delta m_{21}^2$ . The error on the value of  $\sin^2 2\theta_{13}$  is taken to be 0.01. Note that these priors are very conservative. Daya Bay is expected to reduce the error on  $\sin^2 2\theta_{13}$  to 0.005 by 2016. More data from Borexino [45], T2K [3] and NO $\nu$ A [4] will make more precise measurements of solar and atmospheric parameters, before the LENF can be constructed. For the systematics, we have used normalization error of 2% for the signal and 20% for the background. We also include a small tilt of 0.01% for the signal as well as for the background. We have included only the charge mis-identification background, at a level of  $10^{-2}$  for the  $\nu_e$  ( $\bar{\nu}_e$ ) disappearance channels.

### III. RESULTS AND DISCUSSION

We now proceed to analyse the hierarchy determination ability of electron disappearance channel using a low energy neutrino factory. First we fix all the oscillation parameters at their central values reported in section II and set  $\sin^2 2\theta_{13} = 0.113$  and  $\delta_{CP} = 90^\circ$ . Using the standard LENF setup of muon beam energy of 4.5 GeV and a baseline of 1300 km, described in the last section and considering running time of 2 years, we evaluate the number of electron (positron) events that would be observed in the detector. The number of these events resulting from  $\nu_e$  ( $\bar{\nu}_e$ ) interactions as a function of the observed lepton energy, corresponding to each of the hierarchies are displayed in Fig. 1. In our entire study we continue to use the same TASD detector mass, energy resolution, electron detection efficiency, background level and number of useful muons per polarity, mentioned in section II and are as specified in Ref. [41], even when the muon beam energy and baseline length are varied.

The role of systematics and relative value of  $\sin^2 2\theta_{13}$  in the electron disappearance channel had been discussed in section I. We now substantiate our claim, that for the current value of  $\sin^2 2\theta_{13}$ , one can distinguish between the two hierarchies in spite of systematic uncertainties. In Fig. 2, we have plotted the  $e^-$  and  $e^+$  events for NH and IH for  $\sin^2 2\theta_{13} = 0.10$  ( $\approx$  current value) and for  $\sin^2 2\theta_{13} = 0.01$  (a much smaller value). Bands around the event rates denote the combination of errors, statistical and systematic added in quadrature. One can see that for  $\sin^2 2\theta_{13} = 0.01$ , the



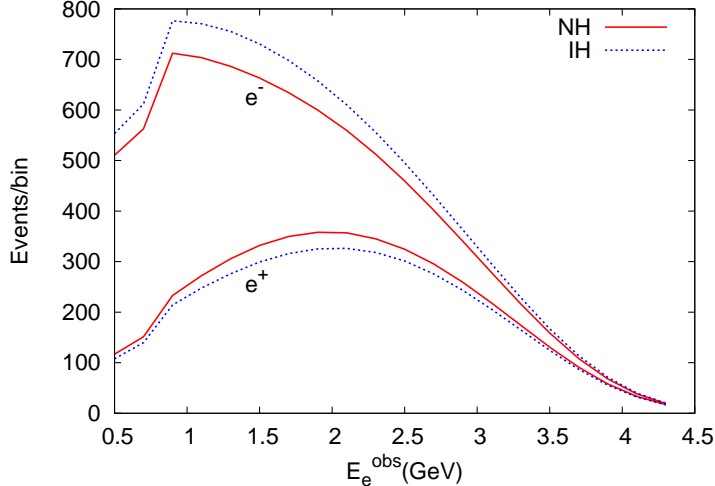


FIG. 1. Electron/positron event rates per bin (0.2 GeV) for  $\sin^2 2\theta_{13} = 0.113$ , as a function of the observed lepton energy at a LENF corresponding to NH (red/solid curve) and IH (blue/dashed curve). The events are for a 2 years exposure of a 20 kton detector at a baseline of  $L = 1300$  km, for neutrinos from 4.5 GeV muon beams with  $1.4 \times 10^{21}$  useful muon decays per year per polarity. The other detector specifications are as mentioned in section II.

errors are larger than the difference between the event rates for the two hierarchies. Even if the exposure and hence statistics is increased by say a factor of 10, reducing the relative statistical error, systematic effects will still result in an overlap between NH and IH. Therefore hierarchy discrimination is not possible. However, for  $\sin^2 2\theta_{13} = 0.10$ , we see that there is enough separation between the events corresponding to the two hierarchies. Hence, with large enough  $1 - 3$  mixing angle, it is possible to go beyond the regime where electron disappearance had been a systematics-riddled channel.

For this LENF setup, we determine the exclusion region in the true value of  $\theta_{13}$  and true value of  $\delta_{CP}$  plane, for which the wrong hierarchy can be excluded at  $3\sigma$  and  $5\sigma$ . Fig. 3 exhibits this region and is to be interpreted as being on the right of the contours. These results have been obtained after marginalization over all the oscillation parameters and shown for both normal and inverted hierarchies. We find that hierarchy can be determined at all values of  $\delta_{CP}$ , leading to the contours being vertical lines. It is clear from Fig. 3 that if the true hierarchy is NH, then above  $\sin^2 2\theta_{13} = 0.112$  the wrong hierarchy, i.e, IH can be excluded at  $5\sigma$ . If IH is the true hierarchy, then NH can be eliminated at  $5\sigma$  for  $\sin^2 2\theta_{13} = 0.111$  and beyond. Also, above  $\sin^2 2\theta_{13} = 0.067$ ,

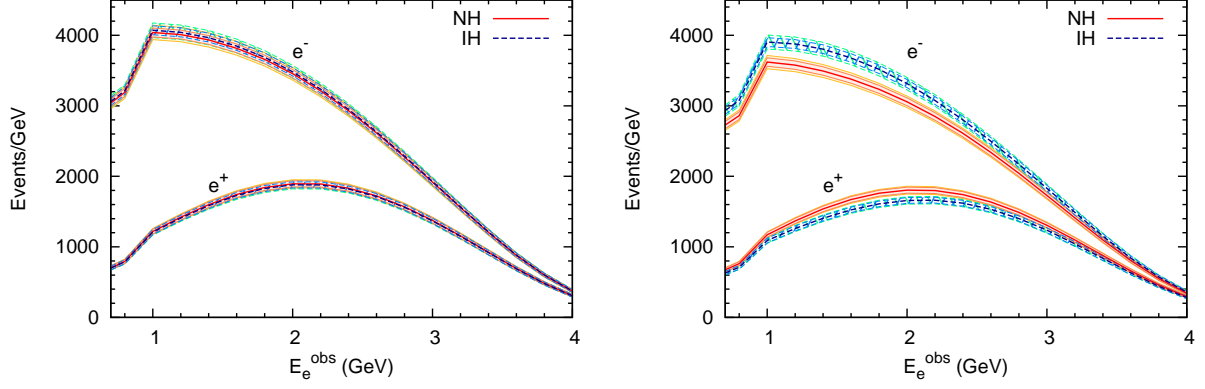


FIG. 2. Electron/positron event rates per bin (1.0 GeV) for  $\sin^2(2\theta_{13}) = 0.01$  (Left) and 0.1 (Right) as a function of the observed lepton energy at a LENF. The bands around each of the curves denote the combination of statistical and systematic errors added in quadrature. All other specifications are similar to that in Fig. 1.

the wrong hierarchy can be excluded at  $3\sigma$ , irrespective of the true hierarchy.

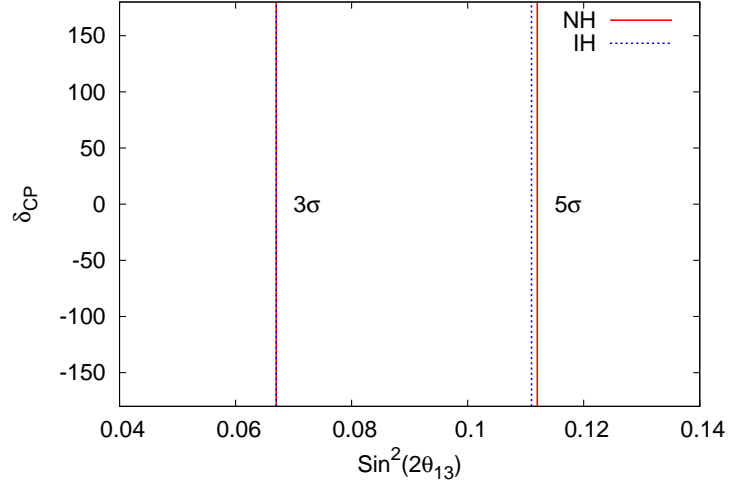


FIG. 3.  $3\sigma$  and  $5\sigma$  Hierarchy exclusion plot at a LENF with 4.5 GeV  $\mu^\pm$  beams (both polarities) over a 2 years exposure with a 20 kton detector located at a baseline of  $L = 1300$  km.

To investigate the hierarchy sensitivity to baselines ( $L$ ), muon beam energy ( $E_\mu$ ), and different detector characteristics such as energy resolution, efficiency etc, requires a complex numerical optimization. In order to find the optimal baseline and the muon beam energy in determining the mass

hierarchy, we assume NH to be the true hierarchy. We fix our  $\sin^2 2\theta_{13}$  in the  $3\sigma$  range of the current best fit value. We have analysed the  $\chi^2$  for different values of baselines in the range  $250 - 2500$  km and for different values of  $E_\mu$ . We have performed this analysis for the LENF setup with a running time of 2 and 5 years for the parent muon beam energy  $E_\mu$  starting at  $2.0$  GeV and going up to  $10.0$  GeV. We keep all oscillation parameters fixed at their central values, while  $\delta_{CP} = 90^\circ$ . We have included the charge mis-identification background at the level of  $10^{-2}$ . In Fig. 4, we use the central value of  $\sin^2 2\theta_{13} = 0.113$  measured by RENO as our true value. The two horizontal lines in the plot indicate the  $\chi^2$  corresponding to  $3\sigma$  and  $5\sigma$ . From this figure one notices that with only 2 years of data, the minimum baselines required for exclusion of the wrong hierarchy at  $5\sigma$ , vary from  $1230$  km to  $1600$  km, corresponding to  $3 - 10$  GeV muon beams and that with increased data from 5 years, the spread in the minimum baselines required reduces appreciably, particularly for higher energies, resulting in baselines in the range  $1120 - 1250$  km being sufficient for a mass hierarchy discrimination for the same  $3 - 10$  GeV range. Thus as long as we have  $E_\mu > 3.0$  GeV and  $L > \sim 1200$  km, hierarchy determination is possible at about  $5\sigma$  using the electron disappearance channel alone.

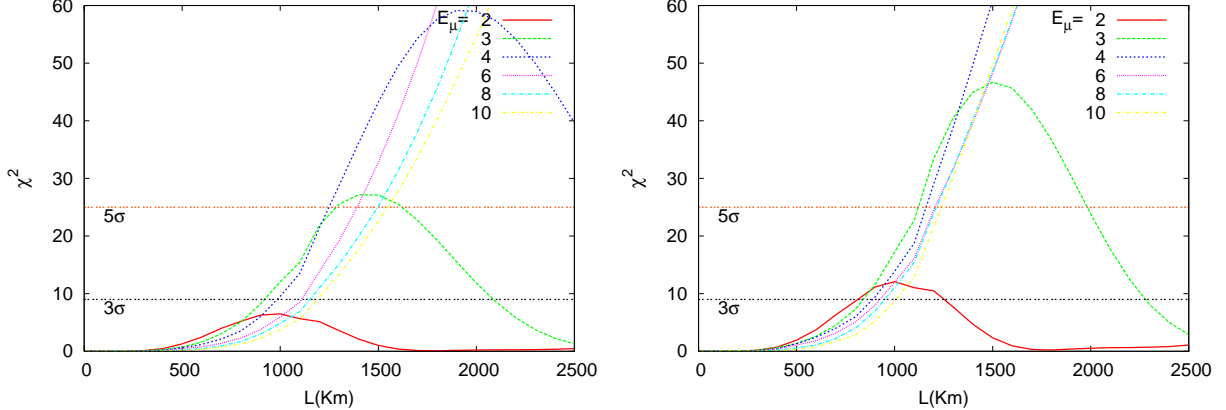


FIG. 4.  $\chi^2$  as a function of the baseline  $L$  for different values of the muon beam energy  $E_\mu$  at a LENF over a 2 years (Left) and 5 years (Right) exposure for input  $\sin^2 2\theta_{13} = 0.113$  and  $\delta_{CP} = 90^\circ$ . We keep all other oscillation parameters fixed at their central values and assume normal hierarchy (NH) to be true hierarchy.

We next plot the contours corresponding to  $3\sigma$  and  $5\sigma$  mass hierarchy discrimination in the  $E_\mu - L$  plane and these are shown in Fig. 5. For these plots we marginalize over all the oscillation parameters. The true value of  $\sin^2 2\theta_{13} = 0.113$  and  $0.078$  have been used in the left panel. While

the first value is the best fit value from RENO, the second smaller value is  $2\sigma$  lower limit of the global fit of the neutrino oscillation parameters. The left figure shows the variation of the contours with change in the magnitude of  $\sin^2 2\theta_{13}$ , whereas the right figure shows the variation for a fixed value with change in exposure. Again, higher exposure is more effective in reducing the minimum baseline than the minimum energy, required for a  $3\sigma/5\sigma$  mass hierarchy discrimination. With 5 years data, beyond  $\sim 5$  GeV, there is negligible change in the minimum baseline needed for a  $5\sigma$  mass hierarchy determination. Moreover, the marginalization of parameters seems to slightly further reduce the spread in minimum baselines for the  $3 - 10$  GeV muon beam energies, at which this measurement is feasible. While we have shown the contours corresponding only to the normal hierarchy being the true one, however, similar results also hold for the case of inverted hierarchy being true, with minor changes in the values of minimum baseline and muon beam energy at which a  $3\sigma/5\sigma$  mass hierarchy discrimination is achieved.

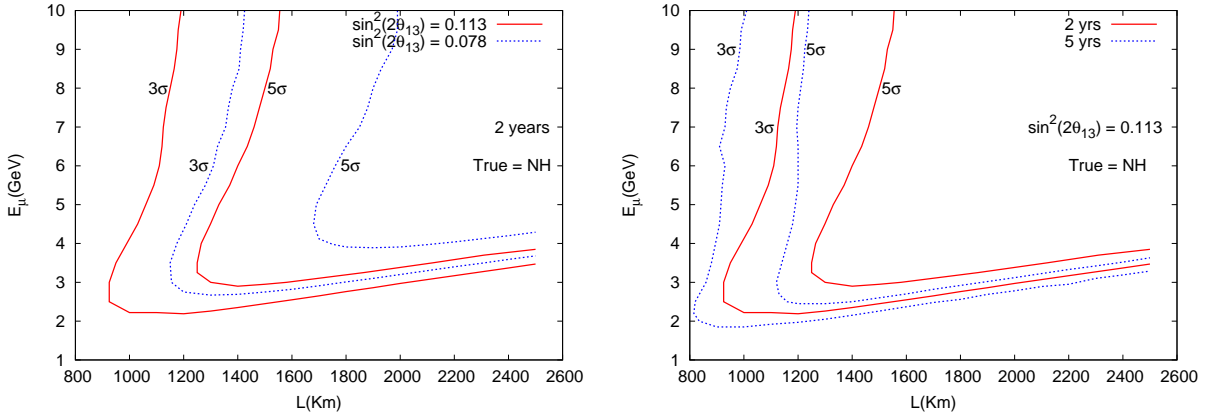


FIG. 5. Left:  $3\sigma$  and  $5\sigma$  contours in  $L - E_\mu$  plane at a LENF over a 2 years exposure for input  $\sin^2(2\theta_{13}) = 0.113$  (red/solid curves) and  $\sin^2(2\theta_{13}) = 0.078$  (blue/dashed curves), respectively. Right:  $3\sigma$  and  $5\sigma$  contours in  $L - E_\mu$  plane at a LENF over a 2 years (red/solid curves) and a 5 years (blue/dashed curves) exposure for input  $\sin^2(2\theta_{13}) = 0.113$ . We have assumed normal hierarchy (NH) to be the true hierarchy.

We would like to mention that since our studies indicate that even with muon beam energies as low as 3 GeV, electron disappearance can be used for mass hierarchy determination, hence in principle, this could also be done at the proposed Very low energy neutrino factory (VLENF) [46], in conjunction with an additional far detector. The VLENF is being proposed as a setup for a short baseline experiment to resolve the several observed anomalies which could possibly arise from

eV-scale sterile neutrinos [47]. In practise however, for measurements at a detector 1200 km away, as the neutrino beam will have to be pointed towards it, this would require that the decay ring for nuSTORM be built on a slope and the near detector located underground, leading to huge additional construction costs [48]. Moreover, the flux at VLENF will be few orders of magnitude smaller than that achievable at a LENF and hence this option does not seem feasible.

#### IV. CONCLUSION

Precision measurement of all the oscillation parameters is the main goal of future neutrino factories. Although some parameters are measured to a very good accuracy, there are still some unknowns such as the Dirac CP phase  $\delta_{CP}$ , neutrino mass hierarchy and the octant of the atmospheric angle  $\theta_{23}$ . Recent measurements by Daya Bay, Double Chooz, and RENO have confirmed a large value of  $\theta_{13}$ . This leads to a rather optimistic scenario for determination of all these unknown parameters.

In this paper, we explore the possibility of using the electron disappearance channel, i.e, the  $\nu_e (\bar{\nu}_e) \rightarrow \nu_e (\bar{\nu}_e)$  oscillation, at a low energy neutrino factory (LENF) to determine the neutrino mass hierarchy. While the  $\nu_e \rightarrow \nu_\mu$  golden channel, has been extensively studied for determination of parameters at LENF, the electron disappearance channel has the advantage of being independent of the other unknowns: CP violating phase  $\delta_{CP}$  and octant of  $\theta_{23}$ . We find that for muon beam energies  $E_\mu > 3.0 \text{ GeV}$  and baselines  $L > 1200 \text{ km}$ , the electron disappearance channel has the capability of neutrino mass hierarchy determination at  $5\sigma$ , for an exposure of a 20 kton totally active scintillating detector to two years of each muon polarity, with  $1.4 \times 10^{21}$  useful muons decays per year.

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